

IX.2. A Semiconductor Device Primer

Bibliography:

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3. Kittel, C., *Introduction to Solid State Physics* (John Wiley & Sons, New York, 1996) QC176.K5, ISBN 0-471-11181-3

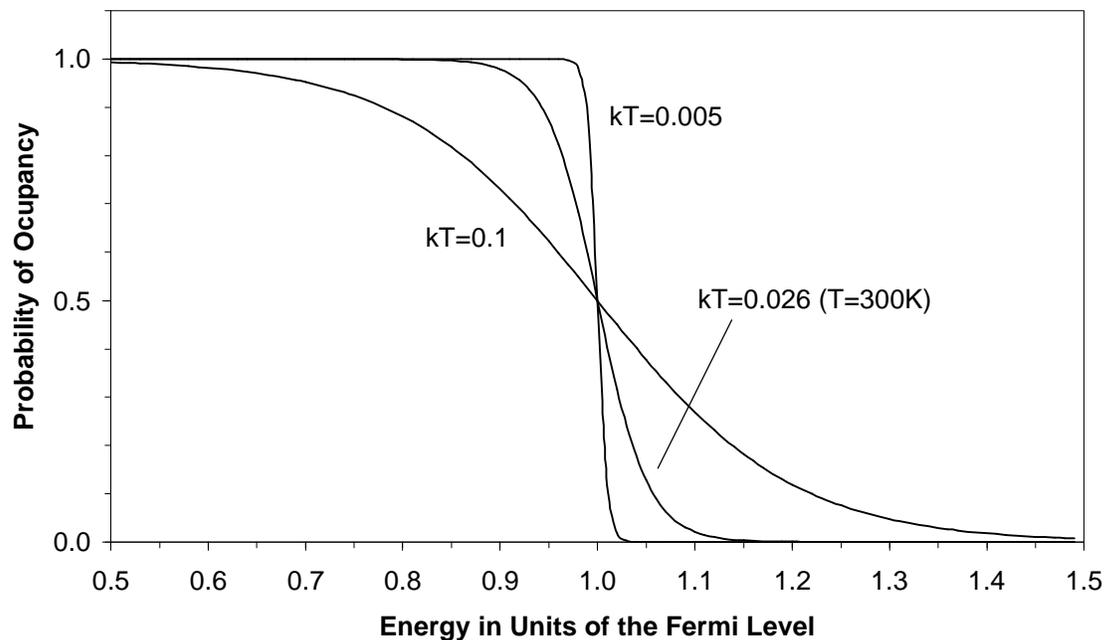
1. Carrier Concentrations

The probability of an electron state in the conduction band being filled is given by the Fermi-Dirac distribution

$$f_e(E) = \frac{1}{e^{(E-E_F)/k_B T} + 1}$$

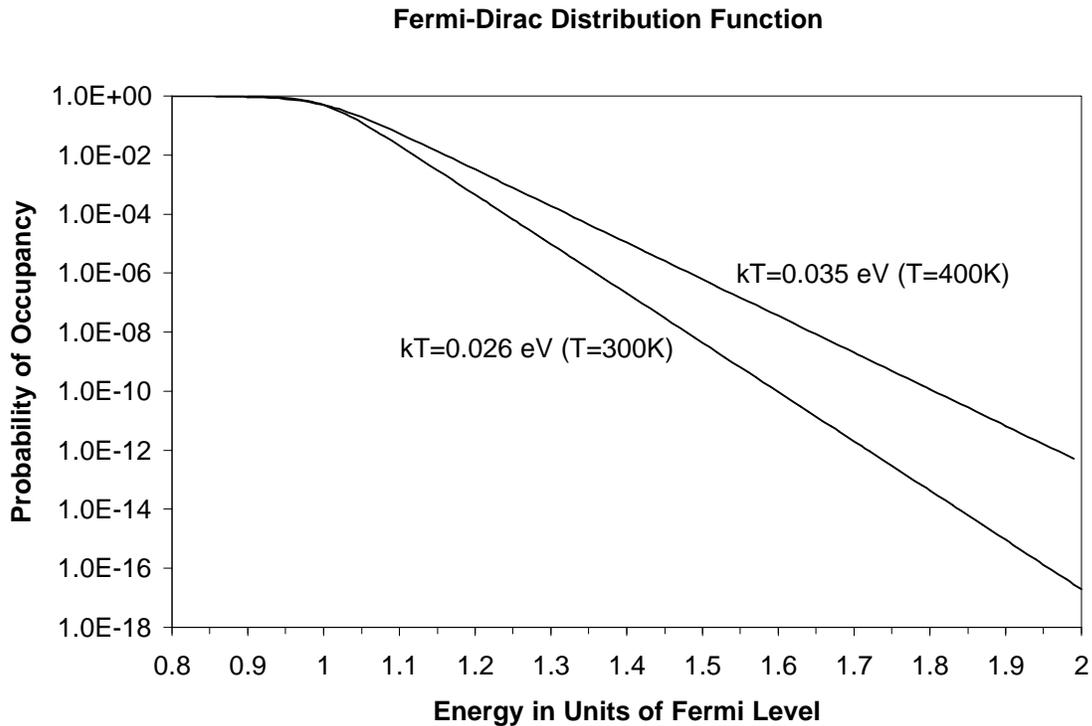
The transition broadens as the thermal energy E increases, as illustrated for $k_B T = 0.005$, $k_B T = 0.026$ and $k_B T = 0.1$ eV.

Fermi-Dirac Distribution Function



The density of atoms in a Si or Ge crystal is about $4 \cdot 10^{22}$ atoms/cm³.

Since the minimum carrier density of interest in practical devices is of order 10^{10} to 10^{11} cm⁻³, very small occupancy probabilities are quite important.



In silicon the band gap is 1.12 eV. If the Fermi level is at midgap, the band-edges will be 0.56 eV above and below E_F .

As is apparent from the plot, relatively large deviations from the Fermi level, i.e. extremely small occupancies, will still yield significant carrier densities

The number of occupied electron states N_e is determined by summing over all available states multiplied by the occupation probability for each individual state

$$N_e = \sum_i m_i f(E_i)$$

Since the density of states near the band edge tends to be quite high, this can be written as an integral

$$N_e = \int_{E_c}^{\infty} f(E)g(E)dE$$

where $g(E)$ is the density of states.

Solution of this integral requires knowledge of the density of states.

Fortuitously, to a good approximation the density of states near the band edge has a parabolic distribution

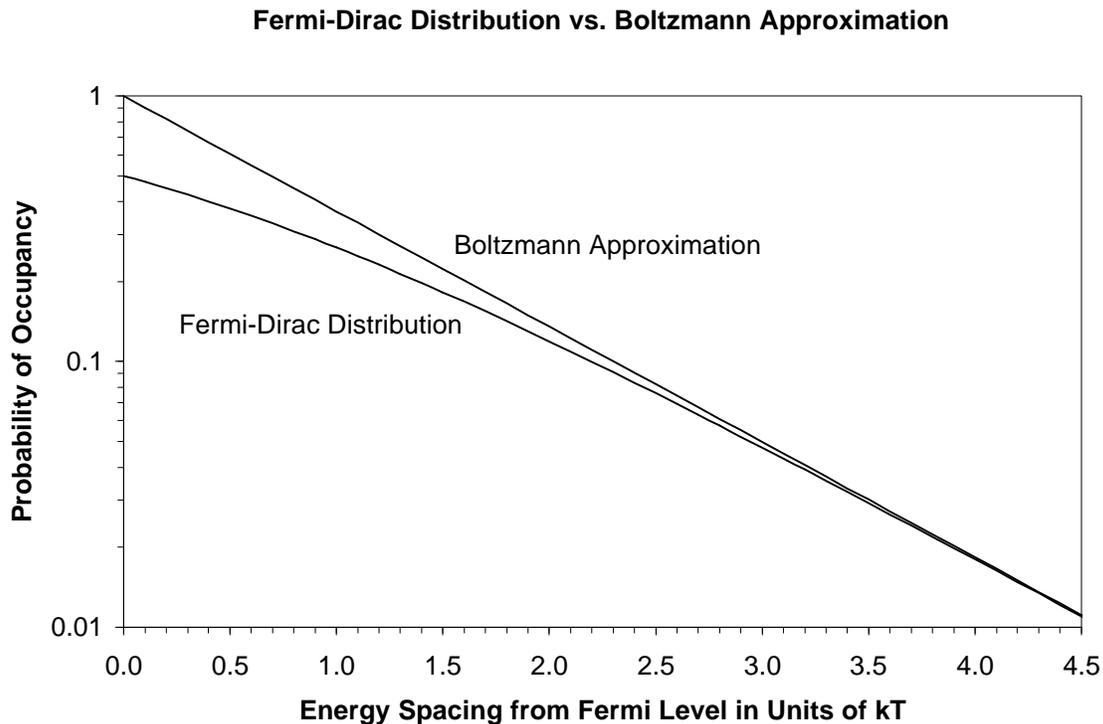
$$g(E)dE \propto (E - E_c)^{1/2}$$

As the energy increases beyond the band edge, the distribution will deviate from the simple parabolic form, but since the probability function decreases very rapidly, the integral will hardly be affected.

The second obstacle to a simple analytical solution of the integral is the intractability of integrating over the Fermi distribution.

If $E-E_F$ is at least several times $k_B T$, the Fermi distribution can be approximated by a Boltzmann distribution

$$1 + e^{(E-E_F)/k_B T} \approx e^{(E-E_F)/k_B T} \Rightarrow f(E) \approx e^{-(E-E_F)/k_B T}$$



At energies beyond $2.3 k_B T$ of the Fermi level the difference between the Boltzmann approximation and the Fermi Distribution is $<10\%$, for energies $>4.5 k_B T$ it is less than 1% .

Applying the approximation to the occupancy of hole states, the probability of a hole state being occupied, i.e. a valence state being empty is

$$f_h(E) = 1 - f_e(E) = \frac{1}{e^{(E_F-E)/k_B T} + 1} \approx e^{-(E_F-E)/k_B T}$$

Since the band gap is of order 1 eV and $k_B T$ at room temperature is 0.026 eV , the conditions for the Boltzmann approximation are fulfilled for excitation across the band gap.

With these simplifications the number of electrons in the conduction band in thermal equilibrium is

$$N_e \propto (k_B T)^{3/2} e^{-(E_c - E_F)/k_B T}$$

and the electron concentration

$$n = N_c e^{-(E_c - E_F)/k_B T}$$

where N_c is the effective density of states at the band edge.

Correspondingly, the hole concentration

$$p = N_v e^{-(E_F - E_v)/k_B T}$$

The effective densities of states in Si and Ge are

$$\begin{array}{ll} \text{Silicon:} & N_c = 2.8 \times 10^{19} \text{ cm}^{-3} \quad N_v = 1.04 \times 10^{19} \text{ cm}^{-3} \\ \text{Germanium:} & N_c = 1.04 \times 10^{19} \text{ cm}^{-3} \quad N_v = 6.0 \times 10^{18} \text{ cm}^{-3} \end{array}$$

In a pure semiconductor each electron in the conduction band corresponds to a hole in the valence band,

$$n = p = n_i$$

where n_i is the carrier concentration intrinsic to an ideal crystal where the only population mechanism is excitation across the band gap.

Using the above results

$$n_i = N_c e^{-(E_c - E_F)/k_B T} = N_v e^{-(E_F - E_v)/k_B T}$$

which one can solve to obtain E_F

$$E_F = \frac{E_c + E_v}{2} - \frac{k_B T}{2} \log(N_c / N_v)$$

If the band structure is symmetrical ($N_c = N_v$), the intrinsic Fermi level lies in the middle of the band gap.

Even rather substantial deviations from a symmetrical band structure will not affect this result significantly, as N_c/N_v enters logarithmically and $k_B T$ is much smaller than the band gap.

$$\begin{array}{ll} \text{Silicon } (E_g = 1.12 \text{ eV}): & n_i = 1.45 \times 10^{10} \text{ cm}^{-3} \text{ at 300K} \\ \text{Germanium } (E_g = 0.66 \text{ eV}): & n_i = 2.4 \times 10^{13} \text{ cm}^{-3} \text{ at 300K} \end{array}$$

For comparison:

The purest semiconductor material that has been grown is Ge with active impurity levels of about $3 \cdot 10^{10} \text{ cm}^{-3}$.

Note that the intrinsic carrier concentration only applies to ideal crystals, where the only source of mobile carriers is thermal excitation across the band gap without any additional impurity atoms or crystal imperfections that would allow other population mechanisms.

A remarkable result is that the product of the electron and hole concentrations

$$np = n_i^2 = N_c N_v e^{-(E_c - E_v)/k_B T} = N_c N_v e^{-E_g/k_B T}$$

depends only on the band gap E_g and not on the Fermi level.

- This result, the law of mass action, is very useful in semiconductor device analysis. It requires only that the Boltzmann approximation holds, so it is not limited to undoped or ideal crystals.

Qualitatively, it says that if one carrier type exceeds the equilibrium concentration, recombination will decrease the concentrations of both electrons and holes to maintain $np = n_i^2$.

2. Carrier Concentrations in Doped Crystals

The equality

$$n_e = n_h$$

only holds for pure crystals, where all of the electrons in the conduction band have been thermally excited from the valence band.

In practical semiconductors the presence of impurities tips the balance towards either the electrons or holes.

Impurities are an unavoidable byproduct of the crystal growth process, although special techniques can achieve astounding results – notably “ultrapure” Ge where the net impurity concentration is about 10^{10} cm^{-3} , i.e. a relative concentration of 10^{-12} .

In semiconductor device technology impurities are introduced intentionally to control the conductivity of the semiconductor.

Let N_d^+ be the concentration of ionized donors and N_a^- the concentration of ionized acceptors.

Overall charge neutrality is preserved, as each ionized dopant introduces a charged carrier and an oppositely charged atom, but the net carrier concentration is now

$$\Delta n = n - p = N_d^+ - N_a^-$$

or

$$p + N_D^+ = n + N_A^-$$

Assume that the activation energy of the donors and acceptors is sufficiently small so that they are fully ionized

$$N_D^+ \approx N_D \quad \text{and} \quad N_A^- \approx N_A$$

Then

$$p + N_D = n + N_A ,$$

which, using $np = n_i^2$, becomes

$$p + N_D = \frac{n_i^2}{p} + N_A$$

If the acceptor concentration $N_A \gg N_D$ and $N_A \gg n_i$

$$\frac{p}{N_A} + \frac{N_D}{N_A} = \frac{n_i}{p} \frac{n_i}{N_A} + 1 \quad \Rightarrow \quad p \approx N_A, \quad n \approx \frac{n_i^2}{N_A} \ll N_A$$

i.e. the conductivity is dominated by holes.

Conversely, if the donor concentration $N_D \gg N_A$ and $N_D \gg n_i$ the conductivity is dominated by electrons.

If the conductivity is dominated by only one type of carrier, the Fermi level is easy to determine. For example, if $n \gg p$, the expression

$$p + N_D = n + N_A$$

can be written

$$n = N_D - N_A$$

$$N_c e^{-(E_c - E_F)/k_B T} = N_D - N_A$$

yielding

$$\frac{E_c - E_F}{k_B T} = \log \left(\frac{N_c}{N_D - N_A} \right)$$

If $N_D \gg N_A$, then $E_c - E_F$ must be small, i.e. the Fermi level lies close to the conduction band edge.

In reality the impurity levels of common dopants are not close enough to the band edge for the Boltzmann approximation to hold, so the calculation must use the Fermi distribution and solve numerically for E_F . Nevertheless, the qualitative conclusions derived here still apply.

It is often convenient to refer all of these quantities to the intrinsic level E_i , as it accounts for both E_c and E_v . Then

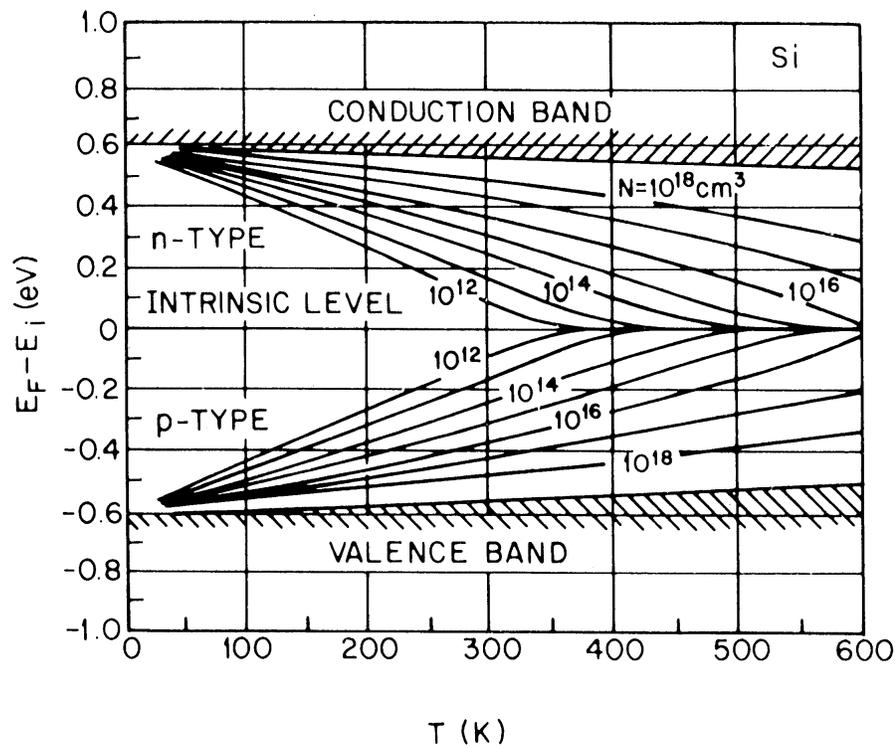
$$n = N_c e^{-(E_c - E_F)/k_B T} = n_i e^{(E_F - E_i)/k_B T}$$

$$p = N_v e^{-(E_F - E_v)/k_B T} = n_i e^{(E_i - E_F)/k_B T}$$

and the Fermi level in the n -region

$$E_F - E_i = -k_B T \log \frac{N_A - N_D}{n_i}$$

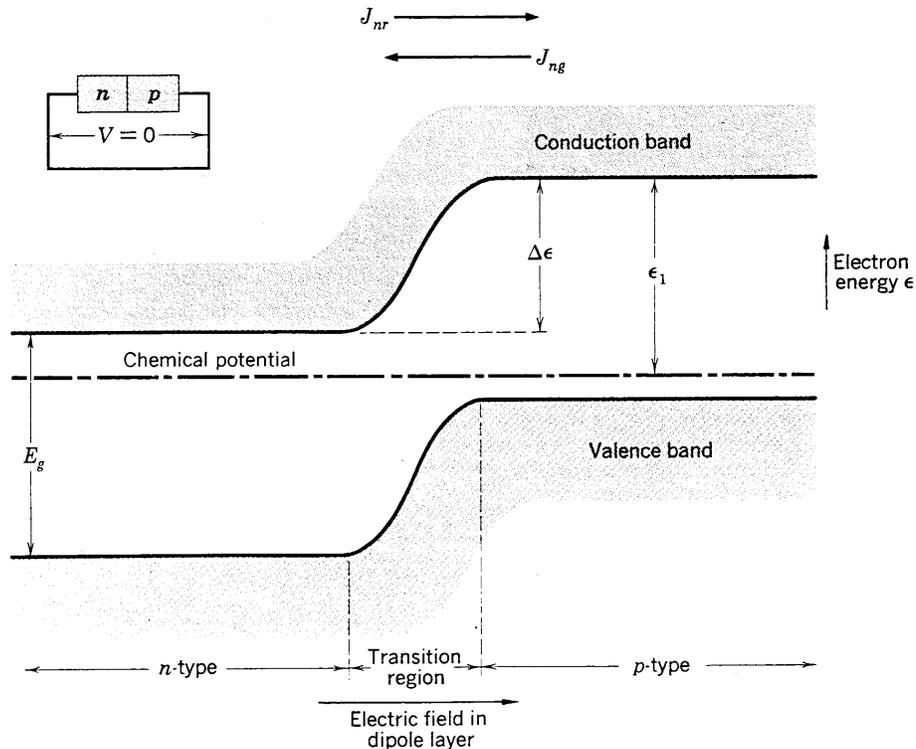
Variation of Fermi level with doping and temperature, including narrowing of the band gap with temperature:



(from Sze)

3. p - n Junctions

A p - n junction is formed at the interface of a p - and an n -type region.



(from Kittel)

Since the electron concentration in the n -region is greater than in the p -region, electrons will diffuse into the p -region.

Correspondingly, holes will diffuse into the n -region.

As electrons and holes diffuse across the junction, a space charge due to the ionized donor and acceptor atoms builds up. The field due to this space charge imposes a velocity component in the opposite direction.

The situation is dynamic:

The concentration gradient causes a continuous diffusion current to flow.

The field due to the space charge drives a drift current in the opposite direction.

Equilibrium is attained when the two currents are equal, i.e. the sum of the diffusion and drift currents is zero.

The net hole current density is

$$J_p = -q_e D_p \frac{dp}{dx} + q_e p \mu_p E_p$$

where D_p is the diffusion constant for holes and E_p is the electric field in the p -region.

To solve this equation we make use of the following relationships:

The hole concentration is

$$p = n_i e^{(E_i - E_F)/k_B T},$$

so its derivative

$$\frac{dp}{dx} = \frac{p}{kT} \left(\frac{dE_i}{dx} - \frac{dE_F}{dx} \right)$$

Since the force on a charge q_e due to an electric field E is equal to the negative gradient of the potential energy,

$$q_e E = -\frac{dE_c}{dx} = -\frac{dE_v}{dx} = -\frac{dE_i}{dx}$$

As only the gradient is of interest and E_c , E_v and E_i differ only by a constant offset, any of these three measures can be used. We'll use the intrinsic Fermi level E_i since it applies throughout the sample.

The remaining ingredient is the Einstein relationship, which relates the mobility to the diffusion constant

$$\mu_p = \frac{q_e D_p}{k_B T}$$

Using these relationships the net hole current becomes

$$J_p = q_e p \frac{D_p}{k_B T} \frac{dE_F}{dx} = \mu_p p \frac{dE_F}{dx}$$

Accordingly, the net electron current

$$J_n = -q_e n \frac{D_n}{k_B T} \frac{dE_F}{dx} = -\mu_n n \frac{dE_F}{dx}$$

Since, individually, the net hole and electron currents in equilibrium must be zero, the derivative of the Fermi level

$$\frac{dE_F}{dx} = 0$$

⇒ in thermal equilibrium the Fermi level must be constant throughout the junction region.

For the Fermi level to be flat, the band structure must adapt, since on the p -side the Fermi level is near the valence band, whereas on the n -side it is near the conduction band.

If we assume that the dopants are exclusively donors on the n -side and acceptors on the p -side, the difference in the respective Fermi levels is

$$\Delta E_F = -k_B T \log \frac{N_A N_D}{n_i^2}$$

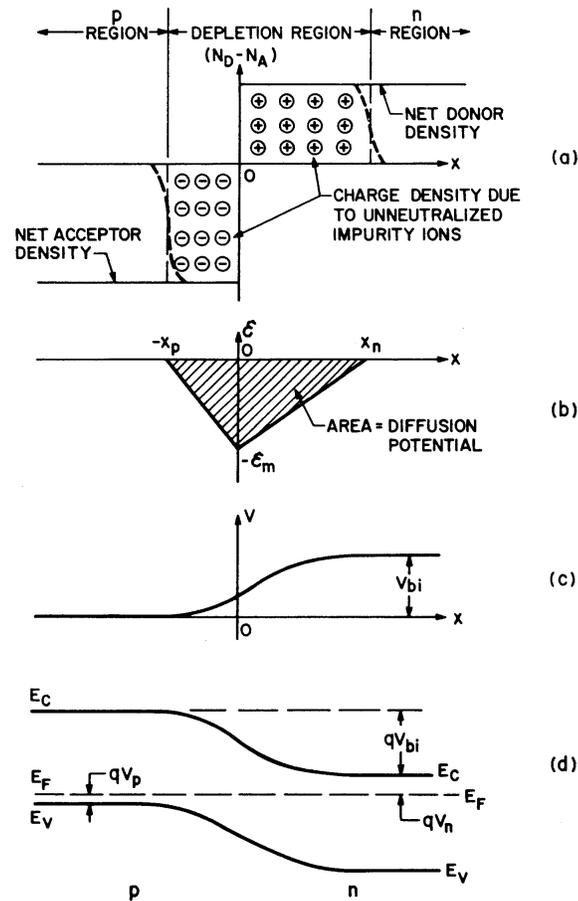
This corresponds to an electric potential

$$\Delta V_F = \frac{1}{q_e} \Delta E_F \equiv V_{bi}$$

often referred to as the “built-in” voltage of the junction.

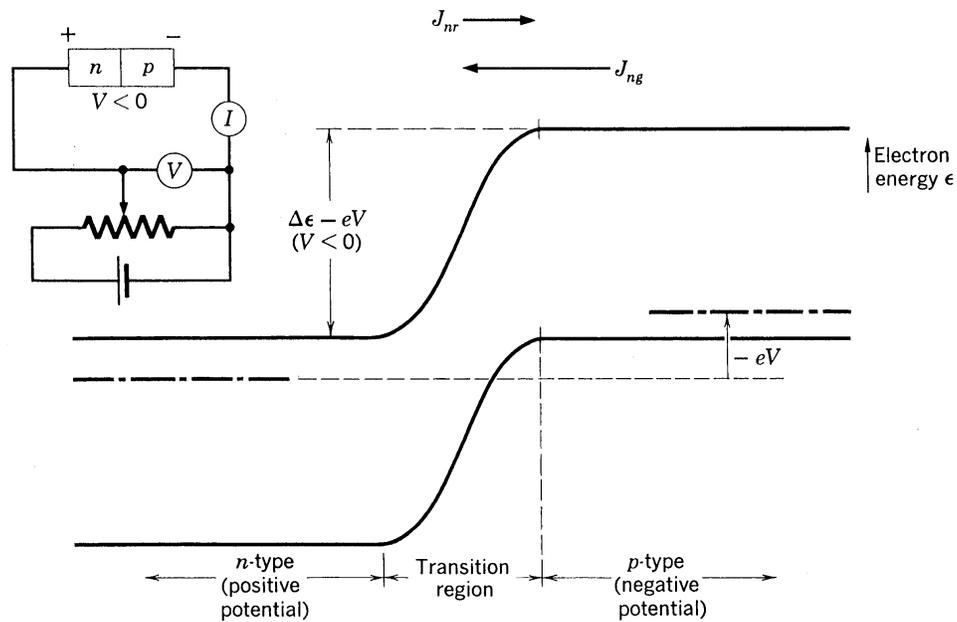
As either N_A or N_D increases relative to n_i , the respective Fermi level moves closer to the band edge, increasing the built-in voltage.

With increasing doping levels the built-in voltage approaches the equivalent potential of the band-gap E_g/q_e .



(from Sze)

The inherent potential distribution in the junction leads to a depletion region, whose width can be increased by application of an external potential, i.e. reverse biasing the junction.



(from Kittel)

This was discussed in a previous lecture.

Now the forward bias mode will be treated in more detail.

Complication:

Applying an external bias leads to a condition that deviates from thermal equilibrium, i.e. the Fermi level is no longer constant throughout the junction.